Estimation in a Linear Model with Serially Correlated Errors When Observations Are Missing

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Abstract This paper compares the asymptotic efficiency of a number of two step estimators developed for estimating a static linear regression model with serially correlated errors when observations are missing. A Monte Carlo simulation is used to illustrate the results in small samples.

1. INTRODUCTION

The presence of serially correlated errors in a linear regression model causes the ordinary least squares (OLS) estimator of the regression coefficients to be inefficient and the OLS formula standard errors to be inconsistent estimates of the true standard errors. To remedy the inefficiency problem, many estimators have been suggested including the maximum likelihood estimator and various two step estimators (Cochrane and Orcutt (1949) and Prais and Winsten (1954)). A consistent estimate of the true standard errors of the OLS estimator could be computed using the Newey-West (1987) approach.

Less attention has been devoted to this problem where for some reason observations on some of the explanatory and dependent variables are missing. OLS could be applied to those observations for which data on all the explanatory variables and the dependent variable are available, and a consistent estimate of the true standard errors of the OLS estimator could be again be computed using the Newey-West (1987) approach. Maximum likelihood (ML) estimates could be obtained by using the Kalman filter algorithm to calculate the log-likelihood function (Harvey and Phillips (1979) and Jones (1980)). This algorithm is used in SAS to allow ML estimation of a static linear regression model with errors that are generated by a pth order autoregression and can handle a wide variety of missing observations (SAS Institute (1984, ch. 9)). SHAZAM will calculate ML estimates for the linear regression model with an autoregression of order one but there can only be one block of missing observations (White and Horsman (1985)).

These ML methods are not available in other computer packages and are computationally expensive. Wansbeek and Kapteyn (WK) (1985) developed a number of simple and easily computable estimators for the parameters of a static linear regression model with first order serially correlated errors when some observations are missing. Their estimators only require estimation by ordinary least squares. Using a Monte Carlo simulation, WK found that, *in small samples*, of the eight estimators they considered the maximum likelihood estimator was "the most complicated but also the most efficient one" (p. 486). However, they did not explicitly derive the asymptotic properties of most of their estimators.

The purpose of the paper is to demonstrate that for the regression parameters, apart from the maximum likelihood estimator, two of the estimators considered by WK are always asymptotically efficient while the asymptotic efficiency of another four depends on the pattern of the missing observations. After defining the regression model and the estimators in section 2, the asymptotic relationships between WK's estimators and two-step asymptotically efficient estimators are indicated in section 3. In section 4, some results from a Monte Carlo experiment are used to indicate whether these efficiency differences are important in small samples. Section 5 contains some concluding comments.

2. THE MODEL AND ESTIMATORS

The model considered is the static linear regression model which can be written for the ith observation as:

$$y_i = X_i\beta + u_i, \qquad (1)$$

where y_i is a scalar, X_i is a 1xk vector of exogenous explanatory variables, β is a kx1 vector of parameters to be estimated, and i=1,..,n. The disturbance u_i is assumed to follow a first-order autoregressive process

$$u_i = \rho u_{i-1} + e_i, \quad -1 < \rho < 1 \qquad (2)$$

where e_i is assumed to be normally and independently distributed with zero mean and variance σ_e^2. The initial u_1 is assumed to be normally distributed with zero mean and variance $\sigma^2 = \sigma_e^2/(1-\rho^2)$.

When there are no missing observations, in addition to the ML estimator, there are several two-step estimators available. The two step estimators are typically based on the Cochrane-Orcutt transformation applied to (1), which for i=2,...,n gives

$$y_i^*(\rho) = X_i^*(\rho)\beta + e_i, \quad (3)$$

where $y_i^*(\rho) = y_i - \rho y_{i-1}$ and $X_i^*(\rho) = X_i - \rho X_{i-1}$. For the first observation, the appropriate transformation is

$$\phi y_1 = (\phi X_1)\beta + \phi u_1, \quad (4)$$

where $\phi = \sqrt{1-\rho^2}$. If ρ was observed, the application of OLS to (3) (or (3) and (4)) could be used to produce a consistent and asymptotically efficient estimate of β since e_i is serially uncorrelated and homoscedastic. Given a consistent estimate of ρ, $\hat{\rho}$, (3) and (4) can be rewritten as

$$y_i^*(\hat{\rho}) = X_i^*(\hat{\rho})\beta + e_i + v_i, \quad (5)$$

$$\hat{\phi} y_1 = (\hat{\phi} X_1)\beta + \hat{\phi} u_1 \quad (6)$$

where $\hat{\phi} = \sqrt{1-\hat{\rho}^2}$, $v_i = u_{i-1}(\beta)(\rho - \hat{\rho})$, and $u_i(\beta) = y_i - X_i\beta$. Application of OLS to (5) [(5) and (6)] is known as the non-iterated version of the Cochrane-Orcutt (1949) [Prais-Winsten (1954)] procedure. Both procedures will give consistent and asymptotically efficient estimates of β.

Suppose now that out of the n possible observations only m $(\leq n)$ are actually observed. The simplest estimator of β in this case is to apply OLS to all the available data to give $\hat{\beta}$. Denote the rank number of the ith actual observation in the original set of observations by n_i with $n_1=1$ and $n_m=n$, and define $t_i = n_i - n_{i-1}$ for $i \geq 2$, the set $I = \{i: 2 \leq i \leq m \text{ and } t_i = 1\}$ which picks out those values of i for which the preceding values of y and X are not missing, and $\bar{I} = \{i: 2 \leq i \leq m \text{ and } t_i > 1\}$ which picks out those values of i for which the preceding values of y or X are missing. For observations in I, (3) is the appropriate transformation to achieve serially uncorrelated errors. For $t_i \geq 1$, repeated substitution of (2) into itself gives

$$u_{n_i} = \rho^{t_i} u_{n_{i-1}} + \eta_{n_i}, \quad (7)$$

where $\eta_{n_i} = \sum_{j=0}^{t_i-1} \rho^j e_{n_i-j}$ and $\text{Var}(\eta_{n_i}) = \sigma_e^2 \sum_{j=0}^{t_i-1} \rho^{2j} = \sigma_e^2 \lambda(\rho)^2$.

Then, defining

$$y_{n_i}^{\#}(\rho) = (y_{n_i} - \rho^{t_i} y_{n_{i-1}}) / \lambda(\rho)$$

$$X_{n_i}^{\#}(\rho) = (X_{n_i} - \rho^{t_i} X_{n_{i-1}}) / \lambda(\rho),$$

and substituting (1) into (7), gives for $t_i \geq 1$,

$$y_{n_i}^{\#}(\rho) = X_{n_i}^{\#}(\rho)\beta + \varepsilon_{n_i}. \quad (8)$$

The error term in (8), $\varepsilon_{n_i} = \eta_{n_i} / \lambda(\rho)$, is serially uncorrelated and homoscedastic. Given a consistent estimate of ρ, $\hat{\rho}$, (8) can be rewritten as

$$y_{n_i}^{\#}(\hat{\rho}) = X_{n_i}^{\#}(\hat{\rho})\beta + \varepsilon_{n_i} + \eta_{n_i} \quad (9)$$

where $\eta_{n_i} = u_{n_{i-1}}(\beta)(\rho^{t_i} - \hat{\rho}^{t_i})$.

For a given initial estimate of ρ, Wansbeek and Kapteyn (1985) suggest the following two step estimators of β:

(i) applying OLS to (9) for $i \in I$;
(ii) applying OLS to (9) for $i \geq 2$; and
(iii) applying OLS to (9) for $i \geq 2$ and (6).

These estimators all require an initial consistent estimate of ρ. Wansbeek and Kapteyn (1985) suggest two simple alternative estimators of ρ:

$$\rho_{CO} = \sum_{i \in I} u_i(\hat{\beta}) u_{i-1}(\hat{\beta}) / \sum_{i \in I} u_{i-1}^2(\hat{\beta})$$

$$\rho_{PW} = \sum_{i \in I} u_i(\hat{\beta}) u_{i-1}(\hat{\beta}) / \left[\sum_{i \in I} u_{i-1}^2(\hat{\beta}) - u_1^2(\hat{\beta}) \right]$$

so that $|\rho_{CO}| \leq |\rho_{PW}|$. The estimator ρ_{CO} is obtained as the regression coefficient from regressing $u_i(\hat{\beta})$ on $u_{i-1}(\hat{\beta})$ when adjacent observations are available.

The combination of the three ways of estimating β and two methods of estimating ρ give rise to six combinations which are labelled by WK (1975, p. 476) as

β_{COCO} : (i) with ρ_{CO},
β_{COPW} : (ii) with ρ_{CO},
β_{COMA} : (iii) with ρ_{CO},
β_{PWCO} : (i) with ρ_{PW},
β_{PWPW} : (ii) with ρ_{PW},
β_{PWMA} : (iii) with ρ_{PW}.

3. ASYMPTOTIC PROPERTIES OF THE ESTIMATORS

It is important to establish the asymptotic properties of the six two step estimators discussed in section 2. This provides a valuable backdrop to any Monte Carlo simulation. There are two ways to establish the asymptotic properties of these estimators. One is to use the results for generated regressors in McKenzie and McAleer (1994) and the other is to rely on Rothenberg-Leenders (1964) theorem concerning two-step estimators.

When there are no missing observations, OLS applied to (1) will be, in general, inefficient asymptotically relative to OLS applied to (3) [or (3) and (4)] and the OLS formula standard errors will be inconsistent estimates of the true standard errors. When a consistent estimate of ρ is used and the explanatory variables are exogenous, the same result holds even though the error term in (5), $e_i + v_i$, is serially correlated and heteroscedastic in small samples this is irrelevant asymptotically because $X_i^{\#}(\hat{\rho})$ is uncorrelated with $u_{i-1}(\beta)$ (see McKenzie and McAleer (1994)). The addition of an additional observation [(6)] to an OLS regression is asymptotically irrelevant.

A similar argument applies in the missing observations case. An estimator based on (ii) which uses an initial consistent estimate of ρ will be asymptotically efficient because although the error term in (9), $\varepsilon_{n_i} + \eta_{n_i}$, is serially correlated and heteroscedastic in small samples this is irrelevant asymptotically because $X_{n_i}^{\#}(\hat{\rho})$ is uncorrelated with $u_{n_{i-1}}(\beta)$ (using McKenzie and McAleer (1994, Theorem 1)). Since options (i) and (ii) both involve OLS regressions we know that throwing observations away will not matter if the number of observations thrown away, j=n-m, is small relative to m, more specifically, if $j/m \to 0$ as $m \to \infty$. However, if $j/m \to q \neq 0$ as $m \to \infty$, throwing away observations matters so that option (ii) will be asymptotically efficient relative to option (i). The difference between option (i) and (iii), one observation [(6)], is asymptotically irrelevant. This discussion indicates that β_{COPW} and β_{PWPW} are always asymptotically efficient, and the asymptotic efficiency of $\beta_{COCO}, \beta_{COMA}, \beta_{PWCO}$ and β_{PWMA} depends on how the number of missing observations changes as the total number of observations increases.

Letting $\theta' = (\beta', \rho, \sigma_e^2)$, $L(\theta)$ be (1/m) times the log-likelihood function, $\partial L / \partial \theta$ be the first derivative of L and $I(\theta) = E\{-\partial^2 L / \partial\theta' \partial\theta\}$ be the information matrix, then Rothenberg-Leenders (1964) showed that using an initial consistent estimate of θ, say $\hat{\theta}$, consistent and efficient estimates of θ, say $\tilde{\theta}$, can be constructed as:

$$\tilde{\theta} = \hat{\theta} + I(\hat{\theta})^{-1} \partial L(\hat{\theta}) / \partial\theta. \tag{10}$$

Wansbeek and Kapteyn (1985) have already derived the necessary results to enable this result to be employed. They show that the information matrix is block diagonal between β and (ρ, σ^2) so that to calculate a two-step asymptotically efficient estimate of β, we only need to determine $\partial L / \partial\beta$ and $I_{\beta\beta}(\theta) = E(-\partial^2 L / \partial\beta \partial\beta')$.

Define the matrices A, B, C, D, E, and F as follows

$$A = X_1' X_1 \phi^2, \tag{11}$$

$$B = \sum_{i \in I} X_{n_i}^{\#'}(\rho) X_{n_i}^{\#}(\rho), \tag{12}$$

$$C = \sum_{i \in \bar{I}} X_{n_i}^{\#'}(\rho) X_{n_i}^{\#}(\rho), \tag{13}$$

$$D = X_1' y_1 \phi^2, \tag{14}$$

$$E = \sum_{i \in I} X_{n_i}^{\#'}(\rho) y_{n_i}^{\#}(\rho), \tag{15}$$

$$F = \sum_{i \in \bar{I}} X_{n_i}^{\#'}(\rho) y_{n_i}^{\#}(\rho). \tag{16}$$

Then

$$I_{\beta\beta}(\theta) = [A + B + C] / m\sigma_e^2, \tag{17}$$

and

$$\partial L / \partial\beta = \{[D + E + F] - [A + B + C]\beta\} / m\sigma_e^2 \tag{18}$$

Substituting (17) and (18) into (10) implies that a simple two-step asymptotically efficient estimator for β is:

$$\tilde{\beta} = [\hat{A} + \hat{B} + \hat{C}]^{-1} [\hat{D} + \hat{E} + \hat{F}] \tag{19}$$

where ^ indicates that the quantity is evaluated using a consistent estimate of ρ.

WK's estimators can be viewed as arising from differences in (a) the initial consistent estimates of ρ; and (b) the estimates of the information matrix (and first derivatives of the log-likelihood) employed. Provided the initial estimate of ρ is consistent, the choice of ρ has no effect on the asymptotic efficiency of the two-step estimator of β. Both ρ_{CO} and ρ_{PW} are consistent estimates of ρ so an estimate of based β on (19) using either ρ_{CO} or ρ_{PW} will be consistent and asymptotically efficient.

The Rothenberg and Leenders theorem also holds whenever any asymptotically equivalent estimate of $I(\theta)$, G such that $p\lim(G) = I(\theta)$ or the derivative vector $\bar{c}$, such that $\bar{c} - \partial L(\bar{\theta})/\partial\theta$ is $o_p(m^{-1/2})$ and "-" indicates evaluation at a $\sqrt{m}$ consistent estimate is used in (10) (Pagan (1986, Theorem 7)).

β_{COPW} and β_{PWPW} use $[A+B+C]/m\sigma_e^2$ as the estimate of the information matrix and $\{[D+E+F]-[A+B+C]\beta\}/m\sigma_e^2$ as the estimate of the first derivatives of the likelihood. It follows that β_{COPW} and β_{PWPW} are consistent and asymptotically efficient estimates of β. β_{COMA} and β_{PWMA} use $[A+B]/m\sigma_e^2$ as the estimate of the information matrix and $\{[D+E]-[A+B]\beta\}/m\sigma_e^2$ as the estimate of the first derivatives. β_{COCO} and β_{PWCO} use $B/m\sigma_e^2$ as the estimate of the information matrix and $\{E-B\beta\}/m\sigma_e^2$ as the estimate of the first derivatives. The β_{COMA} and β_{PWMA} estimators, and β_{COCO} and β_{PWCO} estimators differ only in their treatment of the first observation but both exclude the first observation following a gap of missing observations. Obviously, neglecting the first observation, that is, neglecting A in (17), and $A\beta$ and D in (18) is irrelevant asymptotically but, in small samples, the first observation is not irrelevant (see WK's results and section 4).

With the usual assumptions about the regressors, $p\lim[A+B+C]/m$ is a positive definite and finite matrix. The asymptotic effect of neglecting the first observation following a group of missing observations, that is, C in (17) and (F-Cβ) in (18) depends on the ratio of the number of neglected observations to the number of observations. If $j/m \to 0$ as $m \to \infty$, then asymptotically the neglected observations are unimportant. Hence, any algorithm disregarding these observations will be as asymptotically efficient as a similar algorithm that takes account of them. Of course, in small samples, disregarding those observations could be important. If $j/m \to q \neq 0$ as $m \to \infty$, then the neglected observations remain important in large samples. Hence, an asymptotically efficient estimator must take them into consideration. Hence, estimators $\beta_{COCO}, \beta_{COMA}, \beta_{PWCO}$ and β_{PWMA} will be asymptotically efficient if $j/m \to 0$ as $m \to \infty$ but will be asymptotically inefficient relative to the maximum likelihood estimator if $j/m \to q \neq 0$ as $m \to \infty$.

4. MONTE CARLO SIMULATION

We have performed some simulation experiments which analyse an important class of missing observations - the case of regularly unobserved data. These experiments are designed not only to illustrate our theoretical results but also to complement the simulations presented by WK. WK's experiments suggest that there is little to be gained in terms of root mean square error (RMSE) in going from an estimator based on option (iii) to one based on option (ii), that is, they suggest most of the efficiency gains compared to option (i) comes from appropriate treatment of the first observation rather than from appropriate treatment of observations following a gap due to missing observations.

The assumed data generating process is
$y_t = \beta_1 + \beta_2 X_t + u_t,\ u_t = u_{t-1} + e_t$
with X_t being generated as either a trending series:
$X_t = \exp(0.04t) + w_t,$
$w_t \sim NID(0.0.0036)$
or a non-trending series:
$X_{1t} \sim NID(0.0.0625)$.
Each experiment involves 200 replications. Estimation is carried out over two sample sizes, 29 and 64, and for three values of ρ, 0.8, 0.6 and -0.8. The simulation experiments are variations on those carried out by Beach and MacKinnon (1978) and WK. The unusual choice of sample size is to ensure that in all experiments the final observation in the series is not missing. We have employed the estimators discussed in section 3 as well as the

OLS estimator. To generate samples of observations that contain regularly unobserved data we have assumed that two consecutive data points out of every seven are not observed thus mimicking daily data. Given the growing importance of daily data in econometric investigations, the effect of this pattern of missing observations on parameter estimation are of particular importance in applied work.

The selection of a particular day with which the data commences affects the position of the unobserved data points and may affect the finite sample properties of the estimators. Our Monte Carlo experiments cover five cases corresponding to the observation period starting on Monday to Friday of the week as well as the case of no missing data, denoted "Complete". Patterns listed in the tables correspond to the first day of the day of the data, for example, "Monday" denotes a pattern of having five observations, two missing observations, another five observations, etc., "Tuesday" denotes a pattern of having initially four observations, two missing data points, another five data points, etc., with obvious definitions for Wednesday, Thursday and Friday. When the estimate of ρ did not lie between (-1,1) it was set equal to either -0.99999 or 0.99999. For n=29 and $|\rho = 0.8|$, up to 8% and 10% of the estimates based on ρ_{CO} and ρ_{PW} were greater than unity in absolute value.

Tables 1 and 2 contain the RMSEs of β_2 for ρ =0.8 and 0.6 for non-trending and trending data, respectively, for all seven estimators. They show that the problem of information lost by algorithms neglecting an observation after a gap of missing values (that is, COMA, PWMA, COCO, PWCO) is important in even small sample sizes especially when the data is trending. The good performance of the OLS estimator when the data is trending relative to those estimators that neglect the first observation after a gap of missing observations is noteworthy.

5. CONCLUSION

In this paper, we have derived the asymptotic relationships between a number of two step estimators. An important finding is that the asymptotic efficiency of some of their estimators depends crucially on the pattern of missing observations and their relationship to the usable observations.

6. ACKNOWLEDGEMENTS

The authors would like to thank A. Kapteyn, M. McAleer, R.D. Terrell and T. Wansbeek for their helpful comments on an earlier draft. The first author would like to acknowledge the financial support of the University of Western Australia and a travel grant provided by the Japanese Ministry of Education.

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Table 1: RMSEs of Estimators of β_2 (x1000) - Non-Trending Data

n	ρ	Pattern	OLS	COCO	COPW	COMA	PWCO	PWPW	PWMA
29	0.8	Complete	65	39	40	40	39	39	39
		Monday	79	52	46	52	51	46	52
		Tuesday	79	60	53	61	60	52	61
		Wednesday	77	49	48	51	49	48	50
		Thursday	63	56	50	56	55	50	54
		Friday	67	53	52	53	53	52	53
	0.6	Complete	52	41	41	41	41	41	41
		Monday	70	57	52	56	56	52	56
		Tuesday	76	68	63	68	67	63	68
		Wednesday	41	62	59	61	61	59	60
		Thursday	64	66	56	64	64	58	64
		Friday	66	60	57	56	60	57	56
64	0.8	Complete	4.6	2.7	2.7	2.7	2.7	2.7	2.7
		Monday	5.6	3.5	3.3	3.6	3.5	3.3	3.5
		Tuesday	5.3	3.5	3.3	3.5	3.5	3.3	3.5
		Wednesday	6.1	3.4	3.3	3.4	3.4	3.3	3.4
		Thursday	5.3	3.5	3.3	3.5	3.5	3.3	3.5
		Friday	5.7	3.8	3.6	3.8	3.8	3.6	3.8
	0.6	Complete	4.8	3.0	3.4	3.4	3.0	3.3	3.3
		Monday	5.3	3.8	4.2	3.8	4.1	4.6	4.4
		Tuesday	5.2	2.9	3.3	3.4	2.9	3.2	3.3
		Wednesday	4.8	2.9	3.4	3.3	2.9	3.3	3.2
		Thursday	5.2	2.6	3.8	3.2	2.5	3.6	3.1
		Friday	5.0	3.5	3.9	4.2	3.5	3.9	4.2

Table 2: RMSEs of Estimators of β_2 (x1000) - Trending Data

n	ρ	Pattern	OLS	COCO	COPW	COMA	PWCO	PWPW	PWMA
29	0.8	Complete	66	72	63	63	78	62	62
		Monday	67	103	64	77	129	65	80
		Tuesday	66	96	64	73	128	66	79
		Wednesday	65	82	63	69	103	64	76
		Thursday	65	83	63	70	130	66	80
		Friday	66	96	63	68	112	64	70
	0.6	Complete	40	42	39	39	43	39	39
		Monday	39	57	37	49	61	38	49
		Tuesday	38	60	38	45	68	38	47
		Wednesday	38	57	37	45	65	37	47
		Thursday	38	51	37	42	67	38	44
		Friday	38	51	37	41	78	39	44
64	0.8	Complete	9.7	9.7	8.8	8.8	9.8	8.8	8.8
		Monday	9.8	11.3	8.8	9.4	12.4	8.8	9.8
		Tuesday	9.8	11.4	8.9	10.7	12.5	8.9	11.0
		Wednesday	9.7	11.7	8.9	10.5	13.5	8.8	11.1
		Thursday	9.7	11.7	9.0	10.2	15.0	9.1	11.1
		Friday	9.6	11.1	8.8	9.7	12.1	8.8	9.9
	0.6	Complete	5.3	6.0	5.4	5.4	6.0	5.3	5.3
		Monday	4.9	5.4	5.0	5.4	5.7	5.0	5.9
		Tuesday	5.5	4.7	5.6	4.9	4.8	5.5	4.9
		Wednesday	5.8	6.9	5.9	5.9	7.6	5.8	6.0
		Thursday	6.0	7.7	6.1	7.1	7.8	6.1	7.2
		Friday	5.8	8.2	5.8	7.0	8.2	5.8	7.0